647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov. - 2022(Old Course)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one: **(07)**

(1) Define: Equivalence class of an element of poset. If R is an equivalence relation on non-empty set A then prove that:
(i) $[a] = [b] \Leftrightarrow a\,R\,b$ (ii) either $[a] = [b]$ or $[a] \cap [b] = \phi$.

(2) Define: Lattice as a poset. If $(L, \leq)$ is a lattice and $a, b, c \in L$ then prove that:
(i) $b \leq c \implies (a * b) \leq (a * c)$ and $(a \oplus b) \leq (a \oplus c)$
(ii) $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$.

Q.1 (B) Answer any one: **(04)**

(1) Define: Least and greatest elements of a poset. Show that $L = \{1, 2, 3, 30, 42, 210\}$ is a poset but not lattice. Draw the Hasse diagram of (L, D).

(2) Prove that a relation R on a set A is symmetric iff $R = R^{-1}$.

Q.1 (C) Answer any one: **(03)**

(1) Define: Chain. Prove or disprove: $(P(A), \subset)$, where $A = \{a, b\}$ is a chain. Justify your answer.

(2) Define: Lower and upper bound in a poset.
If $(P, \leq)$ is a given poset, where $P = \{1,2,3,4,5,6,10,15,30\}$, $\leq = 'D'$ then for the subset $B = \{2,3\}$ of P, lower and upper bounds of B are _____ and _______

Q.2 (A) Answer any one: **(07)**

(1) Define: Boolean algebra. State and prove De Morgan's laws for the Boolean algebra $(B, *, \oplus, ', 0, 1)$.

(2) If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra, then for any $x_1, x_2 \in B$, Prove that :
(i) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$
(ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

Q.2 (B) Answer any one: **(04)**

(1) Define: Cube array representation of Boolean function. Find the cube array representation of $f(x, y, z) = x + y$.

(2) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_1 + x_2 + x_3$ as the sum of product canonical form.

Q.2 (C) Answer any one: **(03)**

(1) Define: Atoms in Boolean algebra. In usual notation find all the atoms of Boolean algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(2)Write all minterms of Boolean expression consisting two variables.

Q.3 (A) Answer any one. **(07)**

(1) Obtain Cauchy-Riemann conditions for an analytic function $f(z) = u(r, \theta) + iv(r, \theta)$.

(2) If $f(z) = u(x, y) + iv(x, y)$ is a complex function and $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$ are fixed complex constants and $z = x + iy$ is complex variable then prove that
$\lim_{z \to z_0} f(z) = w_0$ iff $\lim_{(x,y) \to (x_0,y_0)} u(x, y) = u_0$
and $\lim_{(x,y) \to (x_0,y_0)} v(x, y) = v_0$.

Q.3 (B) Answer any one. **(04)**

(1) Define: Harmonic Function, Show that $\frac{y}{x^2+y^2}$ is harmonic function and also find its conjugate harmonic function.

(2) Find an analytic function $f(z) = u + iv$ such that $u - v = x + y$.

Q.3 (C) Answer any one. **(03)**

(1) In usual notations prove that $\nabla^2 = 4\frac{\partial^2}{\partial z\,\partial \bar{z}}$.

(2) Define: Entire function, Show that $f(z) = (3x + y) + i(3y - x)$ is an entire function.

Q.4 (A) Answer any one. **(07)**

(1) State Green's theorem. State and prove Cauchy's fundamental theorem.

(2) If C is the circle $|z| = R, R > 1$ described in the counter clockwise direction then show that

$$\left|\int_c \frac{logz}{z^2}dz\right| < 2\pi\left(\frac{\pi + logR}{R}\right)$$

Q.4 (B) Answer any one. **(04)**

(1) Prove that $\left|\int_c \frac{1}{z^4}dz\right| \leq 4\sqrt{2}$, where c is the line segment joining $z = i$ to $z = 1$.

(2) Evaluate: $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$.

Q.4 (C) Answer any one. **(03)**

(1) Prove that $\int_0^{1+i} \bar{z}\, dz = 1$.

(2) Find $\int_c \frac{zdz}{(9-z^2)(z+i)}$, C : $|z|$=2

Q.5 (A) Answer any one. **(07)**

(1) State and prove Cauchy's integral formula for derivatives and deduce that

$f^n(z_0) = \frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}}dz.$

(2) State Liouville's theorem. State and prove fundamental theorem of algebra.

Q.5 (B) Answer any one. **(04)**

(1) Evaluate: $\int_c \frac{sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}dz$, where c: $|z| = 1$.

(2) State and prove Cauchy's inequality.

Q.5 (C) Answer any one. **(03)**

(1) Find: $\int_c \frac{z^3+2}{(z+i)^3}dz \;\; c : |z| = 2$

(2) Evaluate: $\int_c \frac{e^{2z}}{(z-1)^4}dz, \;\; c : |z| = 3$

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